12-12-18. Wednesday.

# 4.3: Plasma Oscillations:-



When electron is displaced from equilibrium position it leaves behind the ions. Separation of particles generate electric field and then due to this E.F ~~(again)~~ electrons can recombine with ions and the process continues.

We have to calcutate the frequency

**Assumptions:-**

i) $KT_e = 0$ (There are no thermal motions).

2) $B = 0$ (There is no magnetic field).

3) Ions are fixed in space in a uniform distribution.

4) Electrons are moving along x-axis only.

5) Plasma system is infinite or very large.

Equation of motion:

$$m_e n_e \frac{d\bar{V}_e}{dt} = -n_e q \bar{E} \quad (1)$$

$\because \bar{E} = \bar{\nabla}\phi$ means wave is longitudinal.

$\because \bar{\nabla} = \bar{\nabla}_x \hat{i}$

$\because n = n_1 e^{i(k.x-\omega t)}$

$\because n_0 =$ equilibrium density

$\because n_1 =$ oscillating density

$m_e = m =$ mass of electrons.

$n_e = n =$ no. density of electrons.

$V_e = V =$ Velocity of electrons

linearizing Eq. (1), we have

$$m(n_0+n_1)\frac{d}{dt}(V_0+V_1) = -\cancel{n}q(E_0+E_1)(n_0+n_1).$$

$$= q(n_0+n_1)\nabla(\phi_0+\phi_1) \quad \because E = -\nabla\phi$$

Date: ________

$n_0 =$ equilibrium number density.
$n_1 =$ oscillating number density.
$V_0 =$ equilibrium velocity.
$V_1 =$ oscillating velocity.
$E_0 =$ equilibrium electric field.
$E_1 =$ oscillating electric field.

Now,

$$\cancel{n_0} m \frac{dv_1}{dt} = +e \cancel{n_0} \nabla \phi_1$$

$$m \frac{dV_1}{dt} = e \nabla \phi_1 \quad \text{--- (2)}$$

Poisson's Equation is

$$\bar{\nabla} \cdot \bar{E} = 4\pi e (n_i - n_e)$$

$$\bar{\nabla} \cdot (E_0 + E_1) = -4\pi e (+n_0 + n_1)$$

$$\bar{\nabla} \cdot \bar{E}_1 = -4\pi e n_1 \quad \left(\begin{array}{l}\text{linearized} \\ \text{Poisson's Eq.}\end{array}\right)$$

$\because n_i \approx 0$
$\because n_i = 0$ because ions are fixed.
$\because n_e = n_0 + n_1$

$$\bar{\nabla} \cdot \bar{E}_1 = -4\pi e n_1 \quad \text{--- (3)} \Rightarrow \bar{\nabla} \cdot (-\bar{\nabla} \phi_1) = -4\pi e n_1 \Rightarrow \nabla^2 \phi_1 = 4\pi e n_1$$

Continuity Equation:

$$\frac{\partial n}{\partial t} + \bar{\nabla} \cdot (nv) = 0$$

$$n = n_0 + n_1$$

$$V = V_0 + V_1$$

So,

$$\frac{\partial}{\partial t}(n_0 + n_1) + \bar{\nabla} \cdot \left((n_0 + n_1)(V_0 + V_1)\right) = 0$$

$$\frac{\partial n_1}{\partial t} + \bar{\nabla}(n_0 V_1) + \bar{\nabla}(n_1 V_0) = 0$$

because $n_0 V_0$, $n_1 V_1$ is non-linear part so we neglect this.

$$\frac{\partial n_1}{\partial t} + n_0 \bar{\nabla} \cdot V_1 + V_1 \bar{\nabla} \cdot n_0 + n_1 \bar{\nabla} \cdot V_0 + V_0 \bar{\nabla} \cdot n_1 = 0$$

$\because V_1 \cdot \bar{\nabla} n_0 = 0$
$\because n_1 \nabla V_0 = 0$
$\because V_0 \bar{\nabla} \cdot n_1 = 0$

$$\frac{\partial n_1}{\partial t} + n_0 \underset{\downarrow}{\bar{\nabla} \cdot V_1} = 0 \quad \text{--- (4)}$$

tells change in velocity w.r.t space & time,

$$\bar{\nabla}\cdot\bar{v}_1 = -\frac{1}{n_0}\frac{\partial n_1}{\partial t}$$

Substituting the values of $\bar{\nabla}\cdot v_1$ & $\nabla^2\phi_1$ in eq. (5)

$$\frac{m\,d}{dt}\left(-\frac{1}{n_0}\frac{\partial n_1}{\partial t}\right) = 4\pi e^2 n_1$$

$$-\frac{\partial^2 n_1}{\partial t^2} = \frac{4\pi n_0 e^2}{m} n_1$$

$$-\frac{\partial^2}{\partial t^2} n_1 e^{i(k\cdot x - \omega t)} = \frac{4\pi n_0 e^2}{m} n_1 e^{i(kx-\omega t)}$$

$$-n_1 \frac{\partial}{\partial t}\left(-i\omega e^{i(kx-\omega t)}\right) = \frac{4\pi n_0 e^2}{m} n_1 e^{i(kx-\omega t)}$$

$$-n_1(-i\omega)(-i\omega)\, e^{i(kx-\omega t)} = \frac{4\pi n_0 e^2}{m} n_1 e^{i(kx-\omega t)}.$$

$$-n_1(i^2\omega^2)e^{i(kx-\omega t)} = \frac{4\pi n_0 e^2}{m} n_1 e^{i(kx-\omega t)}$$

$$\omega^2 \cancel{n_1 e^{i(kx-\omega t)}} = \frac{4\pi n_0 e^2 \cancel{n_1 e^{i(kx-\omega t)}}}{m}$$

$$\omega_{pe}^2 = \omega^2 = \frac{4\pi n_0 e^2}{m}$$

$\omega_{pe}^2$ = electron plasma frequency or langmuir wave.

→ Plasma frequency is only the function of number density.

→ Group velocity of this equation is zero.

→ Phase velocity is also zero because there is no "k".